Optimization of Airfoil Design for Minimizing Vibrations

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Introduction

- This project aims to develop an optimized airfoil design that reduces vibrations.
- Vibrations in Airfoils can increase noise
- Enhancing Airfoil design can lead to reduction in structural fatigue and improve acoustic performance

Objectives

Optimize 1D Beam

Determine the optimal values for

- Young's Modulus (1e11 Pa ≤ E ≤ 2.1e11 Pa)
- Moment of Inertia ($6.67e-5\ m^4 \leq I \leq 2.083e-3 m^4$)
    - Base (b): $0.1m \leq b \leq 0.2m$
    - Height (h): $0.2m \leq h \leq 0.5m$

$$I = \frac{bh^3}{12}$$

- Length ($1\ m \leq L \leq 5\ m$)

Goal: Maximize natural frequency

Implement Genetic Algorithm

A Genetic Algorithm starts with a group of possible solutions to find the best one.

Goal: Use a Genetic Algorithm to determine optimal values for a 1D Beam System

Methodology

```python
# Define the problem
class BeamOptimization(ElementwiseProblem):
    # Young's Modulus in Pa
    # Moment of Inertia in m^4
    # Length in m
    def __init__(self):
        super().__init__(n_var=3,
                         n_obj=1,
                         n_constr=0,
                         xl=np.array([1e11, 6.67e-5, 1]),
                         xu=np.array([2.1e11, 2.083e-3, 5]))
```

This code defines a 1D Beam optimization problem using the 'ElementwiseProblem' class.

- **Variables**: Young's Modulus (E), Moment of Inertia (I) and Length (L).
- **Objective**: Single objective optimization done to maximize the eigenvalues for natural frequency ($f_1(x) = \sqrt{\lambda}$)
- **Bounds**: The upper and lower bounds shown are consequently for each one of the variables mentioned before.

Stiffness Matrix	Mass Matrix
$Ke = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$	$Me = \frac{mL}{420}\begin{bmatrix} 156 & 22L & 54 & -13L \\ 22L & 4L^2 & 13L & -3L^2 \\ 54 & 13L & 156 & -22L \\ -13L & -3L^2 & -22L & 4L^2 \end{bmatrix}$

Calculate Matrix 'A'

Rearrange the original eigenvalue equation to form a new matrix 'A' that combines Mass and Stiffness

- $M^{-1}Kv = \lambda v$

Compute Eigenvalues for matrix 'A'

Using the eig function in numpy.linalg compute the eigenvalues for the matrix 'A'

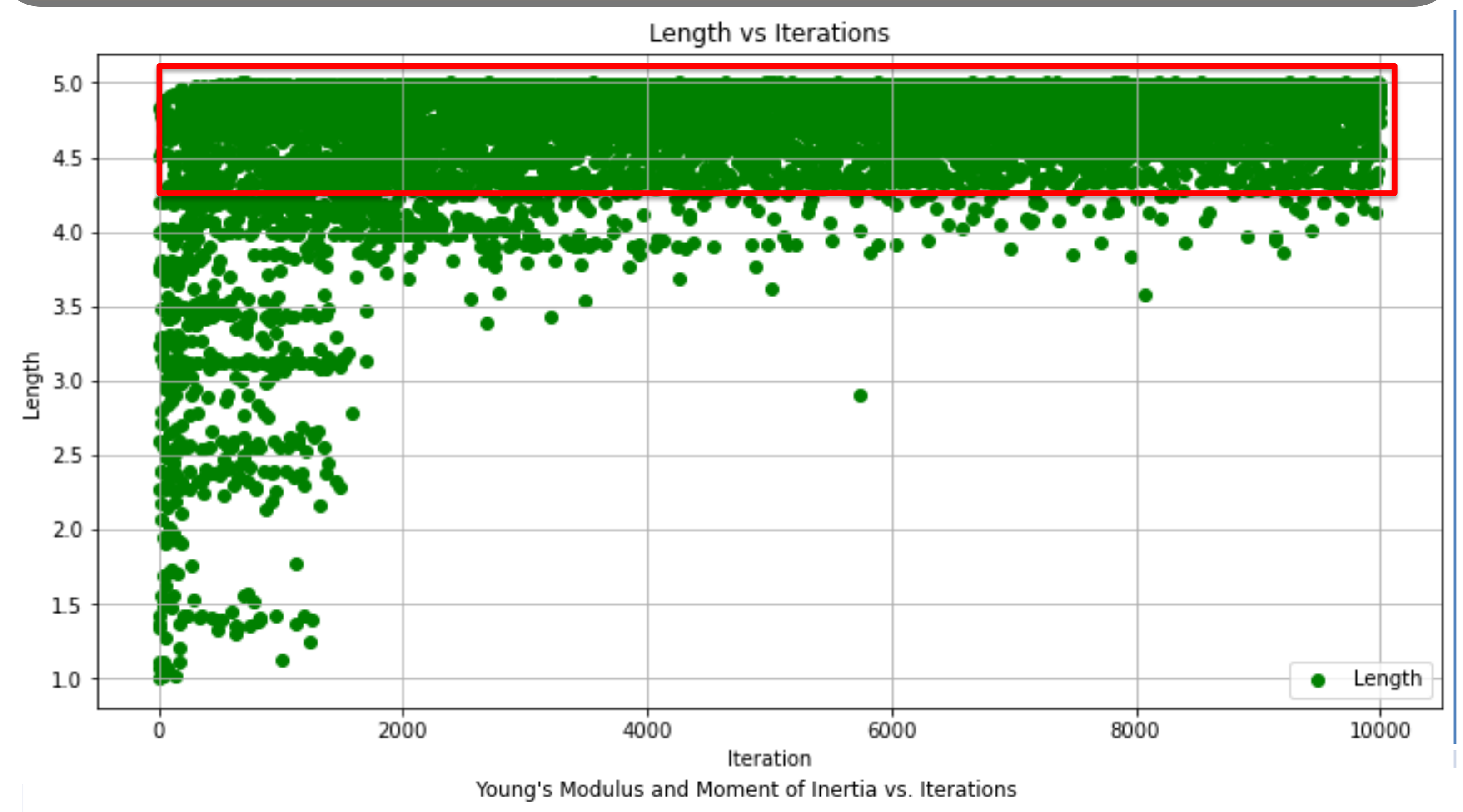
Calculate Natural Frequencies

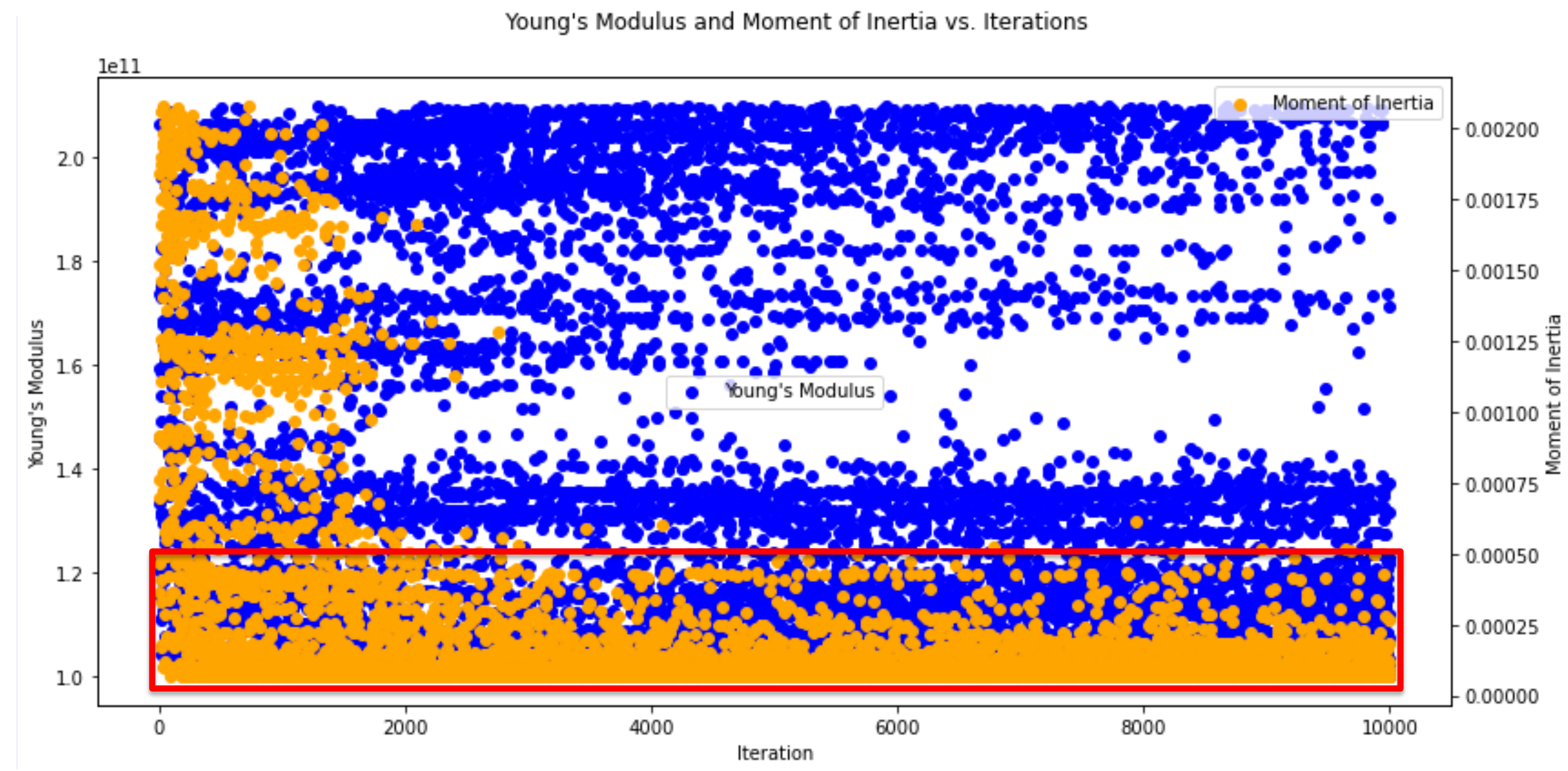
Take the square root of the computed eigenvalues to determine the natural frequencies of the system in Hz

Choosing a Genetic Algorithm (GA):

- **Genetic Algorithms** are well suited for complex optimization problems involving multiple variables with nonlinear relationships.
- **GAs** are also typically used in optimization problems involving one objective

Results / Analysis





Best solution found:

- This output reveals the **optimal parameters** identified by the optimization process.
- These optimal values would typically correspond to a **concentrated cluster** of data points.

E (Young's Modulus) :	I (Moment of Inertia):	L (Length) :
1.202e11	1.655e-3 m^4	3.448 m
Natural Frequencies (Hz):		
Non-rigid body modes	Mode 1: 3.184e4	
	Mode 2 1.088e5	

Analysis:

- Two rigid body modes computed as well due to 2 DOF system without boundary conditions
- The frequencies computed exhibit on broad range

Future Objectives

Enhanced Design Parameters:

- Update the design parameters to include continuous and discrete variables

Broader Design Objectives:

- Include additional design objectives(i.e., fatigue, acoustics)

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References

J. Blank and K. Deb, "Pymoo: Multi-Objective Optimization in Python," in IEEE Access, vol. 8, pp. 89497-89509, 2020